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Polymerized Membranes, a Review

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Abstract

Membranes are of great technological and biological as well as theoretical interest. Two main classes of membranes can be distinguished: Fluid membranes and polymerized, tethered membranes. Here, we review progress in the theoretical understanding of polymerized membranes, i.e. membranes with a fixed internal connectivity. We start by collecting basic physical properties, clarifying the role of bending rigidity and disorder, theoretically and experimentally as well as numerically. We then give a thorough introduction into the theory of self-avoiding membranes, or more generally non-local field theories with δ -like interactions. Based on a proof of perturbative renormalizability for non-local field-theories, renormalization group calculations can be performed up to 2-loop order, which in 3 dimensions predict a crumpled phase with fractal dimension of about 2.4; this phase is however seemingly unstable towards the inclusion of bending rigidity. The tricritical behavior of membranes is discussed and shown to be quite different from that of polymers. Dynamical properties are studied in the same frame-work. Exact scaling relations, suggested but not demonstrated long time ago by De Gennes for polymers, are established. Along the same lines, disorder can be included leading to interesting applications. We also construct a generalization of the $O(N)$ -model, which in the limit $N \rightarrow 0$ reduces to self-avoiding membranes in analogy with the $O(N)$ -model, which in the limit $N \rightarrow 0$ reduces to self-avoiding polymers. Since perturbation theory is at the basis of the above approach, one has to ensure that the perturbation expansion is not divergent or at least Borel-summable. Using a suitable reformulation of the problem, we obtain the instanton governing the large-order behavior. This suggest that the perturbation expansion is indeed Borel-summable and the presented approach meaningful. Some technical details are relegated to the appendices. A final collection of various topics may also serve as exercises.

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Table of Contents

1	Introduction and outline	5
2	Basic properties of membranes	9
2.1	Fluid membranes	9
2.2	Tethered (polymerized) membranes	10
2.3	Crumpling transition, the role of bending rigidity, and some approximations	12
2.4	Stability of the flat phase	14
2.5	Experiments on tethered membranes	19
2.6	Numerical simulations of self-avoiding membranes	21
2.7	Membranes with intrinsic disorder	23
3	Field theoretic treatment of tethered membranes	24
3.1	Definition of the model, observables, and perturbation expansion	24
3.2	Locality of divergences	27
3.3	More about perturbation theory	28
3.4	Operator product expansion (OPE), a pedagogical example	29
3.5	Multilocal operator product expansion (MOPE)	33
3.6	Evaluation of the MOPE-coefficients	35
3.7	Strategy of renormalization	39
3.8	Renormalization at 1-loop order	39
3.9	Non-renormalization of long-range interactions	44
4	Some useful tools and relation to polymer theory	45
4.1	Equation of motion and redundant operators	45
4.2	Analytic continuation of the measure	48
4.3	IR-regulator, conformal mapping, extraction of the residue, and its universality	50
4.4	Factorization for $D = 1$, the Laplace De Gennes transformation	52
5	Proof of perturbative renormalizability	56
5.1	Introduction	56
5.2	Proof	57
5.3	Some examples	70
6	Calculations at 2-loop order	74
6.1	The 2-loop counter-terms in the MS scheme	74
6.2	Leading divergences and constraint from renormalizability	75
6.3	Absence of double poles in the 2-loop diagrams	77
6.4	Evaluation of the 2-loop diagrams	78
6.5	RG-functions at 2-loop order	79

7	Extracting the physical informations: Extrapolations	79
7.1	The problem	79
7.2	General remarks about extrapolations and the choice of variables	80
7.3	Expansion about an approximation	83
7.4	Variational method and perturbation expansion	84
7.5	Expansion about Flory's estimate	85
7.6	Results for self-avoiding membranes	86
8	Other critical exponents and boundaries	87
8.1	Correction to scaling exponent ω	87
8.2	Contact exponents	88
8.3	Number of configurations: the exponent γ	89
8.4	Boundaries	91
9	The tricritical point	92
9.1	Introduction	92
9.2	Double ε -expansion	93
9.3	Results and discussion	97
10	Variants	99
10.1	Unbinding transition	99
10.2	Tubular phase	102
11	Dynamics	103
11.1	Langevin-dynamics, effective field theory	103
11.2	Locality of divergences	106
11.3	Renormalization	107
11.4	Inclusion of hydrodynamic interaction (Zimm Model)	110
12	Disorder and non-conserved forces	113
12.1	The model	115
12.2	Field theoretic treatment of the renormalization group equations	116
12.3	Fluctuation-dissipation theorem and Fokker-Planck equation	117
12.4	Divergences associated with local operators	118
12.5	Renormalization of disorder (divergences associated with bilocal operators)	120
12.6	The residues	122
12.7	Results and discussion	124
12.8	Long-range correlated disorder and crossover from short-range to long-range correlated disorder	128
13	N-colored membranes	129
13.1	The $O(N)$ -model in the high-temperature expansion	130
13.2	Renormalization group for polymers	132
13.3	Generalization to N colors	139
13.4	Generalization to membranes	140
13.5	The arbitrary factor $c(D)$	145

13.6	The limit $N \rightarrow \infty$ and other approximations	145
13.7	Some more applications	147
14	Large orders	151
14.1	Large orders and instantons for the SAM model	152
14.2	The polymer case and physical interpretation of the instanton	155
14.3	Gaussian variational calculation	158
14.4	Discussion of the variational result	160
14.5	Beyond the variational approximation and $1/d$ corrections	164
15	Conclusions	165
A	Appendices	166
A.1	Normalizations	166
A.2	List of symbols and notations used in the main text	167
A.3	Longitudinal and transversal projectors	168
A.4	Derivation of the RG-equations	169
A.5	Reparametrization invariance	171
A.6	Useful formulas	171
A.7	Derivation of the Green function	173
E	Exercises with solutions	174
E.1	Example of the MOPE	174
E.2	Impurity-like interactions	175
E.3	Equation of motion	175
E.4	Tricritical point with modified 2-point interaction	176
E.5	Consequences of the equation of motion	177
E.6	Finiteness of observables within the renormalized model	178
R	References	179

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